SYDNEY TECHNICAL HIGH SCHOOL

YEAR 12 ASSESSMENT TASK 3

JUNE 2003

MATHEMATICS

Time Allowed: 70 minutes

Instructions:

* Attempt all questions.
* Answers to be written on the paper provided.
* Start each question on a new page.
* All necessary working should be shown.
* Marks may not be awarded for careless or badly arranged working.
* This question paper must be stapled on top of your answers.
* Marks shown are for guidance and may be changed slightly if needed.
* Standard integrals are attached and may be removed for your convenience.

Name: ______________________ Teacher: ______________________

Question 1	Question 2	Question 3	Question 4	Question 5	Total
/12	/12	/13	/11	/12	/60

Question 1 (12 marks) **Marks**

a) Change 2.3 radians to degrees and minutes 1

b) Find tan3.2 to 3 decimal places 1

c) Find the exact value of i) $\sin\frac{3\pi}{4}$ 1

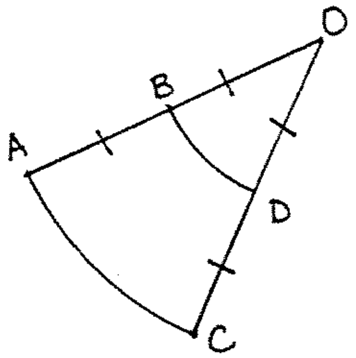
ii) $\cos\left(\frac{-\pi}{6}\right)$ 1

d) Solve $2\sin x = -\sqrt{3}$ *for* $0 \le x \le 2\pi$ 2

e) AC and BD are the arcs of two concentric circles with centre 0.

AB = OB =CD = OD = 10cm.

$\angle AOC = \frac{\pi}{15}$



Find i) the perimeter of Sector OAC in terms of π 2

ii) the area of ABDC in terms of π 2

f) If $f(x) = 2\cos 2x$ find $f'\left(\frac{\pi}{4}\right)$ 2

Question 2 (12 marks)

a) Find i) $\frac{d}{dx}(\sin x^2)$ 1

ii) $\frac{d}{dx}(\tan^2 x)$ 2

b) The gradient function of a curve is given by $\frac{dy}{dx} = 6\cos 2x - \sin x$.

If the curve passes through the origin, find the equation of the curve 3

c) If $y = 3\sin x - 4\cos x$ *show that* $\frac{d^2y}{dx^2} + y = 0$ 3

d) Find $\int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \cos 2x\,dx$ 3

Question 3 (13 marks)

a) Evaluate $\int_0^2 (1-2x)^3\,dx$ 3

b) A curve is defined as follows

$$f(x)=\begin{cases}-1 & for\ -2\le x<0\\ x^2-1 & for\ 0\le x\le 3\end{cases}$$

i) Sketch the curve neatly showing a suitable scale on both axes 2

ii) State the co-ordinates of the point where the curve cuts the x axis. 1

iii) Find the area between the curve, the x axis and the lines x = -2 and x = 3. 4

c) Solve $2\sin^2 x-\sin x=0\ for\ 0\le x\le 2\pi$ 3

Question 4 (11 marks)

a) i) Sketch $y=4\sin\frac{x}{2}$ in the domain $-2\pi\le x\le 2\pi$. State its period and amplitude 3

ii) Hence find the number of solutions to the equation $\sin\frac{x}{2}=\frac{1}{4}$ in the domain $-2\pi\le x\le 2\pi$ 1

b) i) The table below has been completed for $f(x)=\sqrt{\frac{x^2-1}{x}}$

x	1	1.25	1.5	1.75	2
f(x)	0	0.67	0.91	1.09	1.22

ii) By using Simpson's Rule and the five function values in the table find $\int_1^2 f(x)dx$ 2

c) The area bounded by the curve $y = \sqrt{x^3}$, the x axis and the line $x = 3$, is rotated around the x axis. Find the volume of the solid formed (leave your answer in terms of π). 3

d) Differentiate $y = x\sin 2x$ 2

Question 5 (12 marks)

a) i) Find the x values of the points of intersection of the line $y = 2x$ and the curve $3y = x^2$ 1

ii) On the same axes sketch $y = 2x$ and $3y = x^2$, showing clearing the points of intersection. Label the line and the curve and shade the <u>enclosed</u> area. 2

iii) Find the shaded area 2

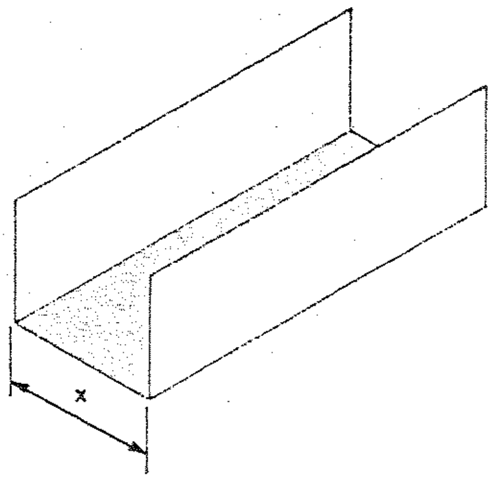
b) A metal gutter open at the top and ends, is bent up from material 30cm wide to form a rectangular cross section.

i) If the base of the gutter is x cm wide, find the height of the gutter. 1

ii) Show that the area of the gutter cross section is given by

$$A = 15x - \frac{x^2}{2.}.$$ 1

iii) Find the value of x for which A is a maximum. 3



c) Find $\int \tan^2\theta \, d\theta$ 2

Question 1 (12 mks)

a) $2.3 \text{ radians} = \underline{131^\circ 47'}$ (1

b) $\tan 3.2 = \underline{0.058}$ (1

c) i) $\sin \frac{3\pi}{4} = \sin\left(\pi - \frac{\pi}{4}\right)$

$= \sin \frac{\pi}{4}$

$= \underline{\frac{1}{\sqrt{2}}}$ (1

ii) $\cos\left(-\frac{\pi}{6}\right) = \cos \frac{\pi}{6}$

$= \underline{\frac{\sqrt{3}}{2}}$ (1

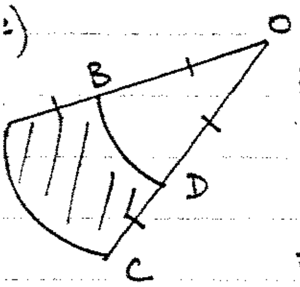
d) $2 \sin x = -\sqrt{3}$

$\sin x = -\frac{\sqrt{3}}{2}$ S | A ✓T | C✓

acute $x = \pi/3$

$\therefore x = \frac{4\pi}{3}, \frac{5\pi}{3}$ (2

e)



i) $P = 40 + 20 \times \frac{\pi}{15}$

$P = \left(40 + \frac{4\pi}{3}\right) \text{ cm}$ (2

ii) $A = \left(\frac{1}{2} \times 20^2 \times \frac{\pi}{15}\right) - \left(\frac{1}{2} \times 10^2 \times \frac{\pi}{15}\right)$

$\therefore A = \frac{1}{2} \times \frac{\pi}{15}\left(20^2 - 10^2\right)$

$\underline{A = 10\pi \text{ cm}^2}$ (2

f) $f(x) = 2\cos 2x$ (2

$f'(x) = -4\sin 2x$

$\therefore f'\left(\frac{\pi}{4}\right) = -4 \sin \frac{\pi}{2}$

$= \underline{-4}$

Question 2 (12 mks)

a) i) $\frac{d}{dx}\left(\sin x^2\right) = \underline{2x \cos x^2}$ (1

ii) $\frac{d}{dx}(\tan x)^2 = 2\sec^2 x . \tan x$ (2

b) $\frac{dy}{dx} = 6\cos 2x - \sin x$

$y = 3\sin 2x + \cos x + c$

sub (0,0)

$0 = 3\sin 0 + \cos 0 + c$

$\therefore c = -1$

$\underline{y = 3\sin 2x + \cos x - 1}$ (3

c) $y = 3\sin x - 4\cos x$

$\frac{dy}{dx} = 3\cos x + 4\sin x$

$\frac{d^2y}{dx^2} = -3\sin x + 4\cos x$

$LHS = \frac{d^2y}{dx^2} + y$

$= (-3\sin x + 4\cos x) + 3\sin x - 4\cos x$

$\underline{= RHS}$ (3

d) $\int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \cos 2x \, dx$ (3

$= \left[\frac{1}{2}\sin 2x\right]_{\frac{\pi}{3}}^{\frac{2\pi}{3}}$

$= \frac{1}{2}\left[\sin \frac{4\pi}{3} - \sin \frac{2\pi}{3}\right]$

$= \frac{1}{2}\left[\sin\left(\pi + \frac{\pi}{3}\right) - \sin\left(\pi - \frac{\pi}{3}\right)\right]$

$= \frac{1}{2}\left[-\sin \pi/3 - \sin \pi/3\right]$

$= \frac{1}{2}\left[-\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}\right] = \underline{-\frac{\sqrt{3}}{2}}$

Question 3 (13 mks)

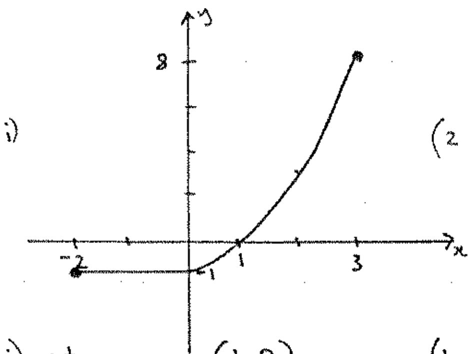
a) $\int_0^2 (1-2x)^3\,dx = \left[\frac{(1-2x)^4}{-8}\right]_0^2$

$= \frac{81}{-8} - \frac{1}{-8}$

$= -10$ (3

b)

i)



(2

ii) pt (1,0) (1

iii) $A = 2 + \left|\int_0^1 (x^2-1)\,dx\right| + \int_1^3 (x^2-1)\,dx$

$= 2 + \left|\left[\frac{x^3}{3}-x\right]_0^1\right| + \left[\frac{x^3}{3}-x\right]_1^3$

$= 2 + \left|\left(\frac{1}{3}-1\right)\right| + \left(6 - -\frac{2}{3}\right)$

$= 2 + \frac{2}{3} + 6\frac{2}{3}$

$= 9\frac{1}{3}$ unit^2 (4

c) $2\sin^2 x - \sin x = 0$

$\sin x(2\sin x - 1) = 0$

$\sin x = 0 \qquad 2\sin x - 1 = 0$

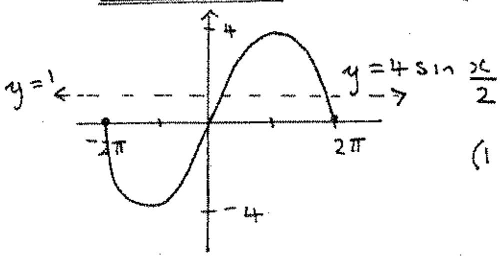
$\sin x = \frac{1}{2}$

$\therefore x = 0, \pi, 2\pi, \frac{\pi}{6}, \frac{5\pi}{6}$ (3

Question 4 (11 mks)

a) i) amp = 4 (1

period = 4π (1



(1

ii) $\sin\frac{x}{2} = \frac{1}{4}$

$4\sin\frac{x}{2} = 1$

see y=1 above ∴

2 solutions (1

b) $\int_1^2 f(x)\,dx =$

$= \frac{\cdot 25}{3}\left[0 + 1\cdot 22 + 4(\cdot 67 + 1\cdot 09) + 2\times\cdot 91\right]$

$= 0\cdot 84$ (2

c) $V_x = \pi\int_0^3 \left(\sqrt{x^3}\right)^2 dx$

$= \pi\int_0^3 x^3\,dx$

$= \pi\left[\frac{x^4}{4}\right]_0^3$

$= \frac{81\pi}{4}$ unit^3 (3

d) $u = x \qquad v = \sin 2x$

$u' = 1 \qquad v' = 2\cos 2x$

$\therefore \frac{d}{dx}(x\sin 2x) = \sin 2x + 2x\cos 2x$ (2

Question 5 (12 mks)

a) i) $y = 2x \qquad 3y = x^2$

$3(2x) = x^2$

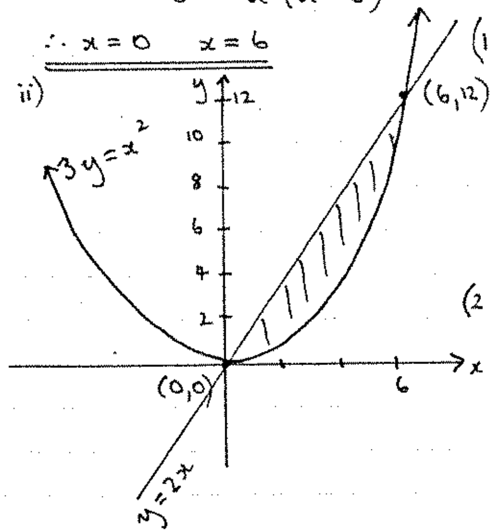
$6x = x^2$

$0 = x^2 - 6x$

$0 = x(x-6)$

$\therefore x = 0 \quad x = 6$ (1

ii)



(2

iii) $A = \int_0^6 \left(2x - \frac{x^2}{3}\right) dx$

$= \left[\frac{2x^2}{2} - \frac{x^3}{9}\right]_0^6$

$= 36 - \frac{216}{9}$

$= 12$ unit^2 (2

c) i) $2h + x = 30$

$2h = 30 - x$

$h = \frac{30-x}{2}$

$\therefore$ height $= \frac{30-x}{2}$ (1

ii) $A = x\left(\frac{30-x}{2}\right)$

$A = 15x - \frac{x^2}{2}$ (1

iii) $\frac{dA}{dx} = 15 - x$

$\frac{d^2A}{dx^2} = -1$

$\therefore$ st pt $\frac{dA}{dx} = 0 \qquad 15 - x = 0$

$x = 15$ (3

when $x = 15$ $\frac{d^2A}{dx^2} < 0$ ∴ max

c) $\int \tan^2\theta\,d\theta$

since $1 + \tan^2\theta = \sec^2\theta$

$\int \tan^2\theta\,d\theta = \int(\sec^2\theta - 1)\,d\theta$

$= \tan\theta - x + c$ (2